4 (1) $u = (x^2+2)$ とおくと、 $y = u^2$ であるため、

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = 2u \times 2x = 2(x^2+2) \times 2x = 4x(x^2+2)$$

(2) $u = (x^3+2x)$ とおくと、$y = 1/u^2$ であるため、

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = -\frac{2}{u^3} \times (3x^2+2) = -\frac{2(3x^2-2)}{(x^3+2x)^3}$$

$$\boxed{\frac{dy}{du} = (u^{-2})' = -2u^{-3} = -\frac{2}{u^3} \ , \ \frac{du}{dx} = (x^3+2x)' = 3x^2+2}$$

(3) $y = \sqrt{(2x+7)} = (2x+7)^{1/2}$

$u = (2x+7)$ とおくと、 $y = u^{1/2}$ であるため、

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = \frac{1}{2}u^{-\frac{1}{2}} \times 2 = u^{-\frac{1}{2}} = (2x+7)^{-\frac{1}{2}} = \frac{1}{\sqrt{2x+7}}$$

(4) $y = \sqrt[3]{(2x+7)^4} = \{(2x+7)^4\}^{\frac{1}{3}} = (2x+7)^{\frac{4}{3}}$

$u = (2x+7)$ とおくと、 $y = u^{\frac{4}{3}}$

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = \frac{4}{3}u^{\frac{4}{3}-1} \times 2 = \frac{4}{3}u^{\frac{1}{3}} \times 2 = \frac{4}{3}(2x+7)^{\frac{1}{3}} \times 2 = \frac{8}{3}\sqrt[3]{(2x+7)}$$

3(3) 別解 $y = \frac{5x}{x^4-x} = \frac{5x}{x(x^3-1)} = \frac{5}{x^3-1} = 5(x^3-1)^{-1}$

$u = (x^3-1)$ とおくと $y = 5u^{-1}$

$$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx} = 5 \times (-1)u^{-2} \times (3x^2) = -5(x^3-1)^{-2} \times 3x^2$$

$$= -\frac{15x^2}{(x^3-1)^2}$$